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Poroacoustic acceleration waves with second sound

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Abstract

A model for coupled acoustic waves and thermal waves in a porous medium is investigated. Due to the use of lighter materials in modern buildings and noise concerns in the environment such models for thermo-poroacoustic waves are of much interest to the building industry. We present a model for acoustic wave propagation in a porous material which also allows for propagation of a thermal wave. The thermodynamics is based on an entropy inequality of A.E. Green and N. Laws, [On the entropy production inequality, *Archive for Rational Mechanics and Analysis* 45 (1972) 47–53]. A fully nonlinear acceleration wave analysis is performed.

1. Introduction

The reduction of noise is a major modern environmental problem and much research effort is directed to studying this, see e.g. Refs. [1,2], and the references therein. In seismic zones buildings are constructed with much lighter porous materials and typically have thinner walls. As a consequence, there is a great need to study the acoustic properties of porous materials including the nature of the solid matrix and the gas filling the pores, and the influence of temperature on these quantities. Measurements are being made of acoustic and related thermal properties of many materials, such as aluminium foams (e.g. Ref. [3]), polyester fibre materials (e.g. Ref. [4]), and models to fit properties have been devised (e.g. Ref. [5]). We observe that in seismic zones, such as the region around Avellino, near Salerno, brick manufacturers typically attempt to increase the porosity (gas volume/total volume) to make the brick lighter. However, this usually has the effect that sound propagation through the brick is amplified and the brick itself becomes more brittle thereby making it less strong when subject to earth movement. In an attempt to create lighter bricks but retain strength, engineering laboratories in the Salerno region are experimenting with filling porous materials used in brick design with small pieces of chemical which when heated infuse into the brick and remain trapped in the pores in gaseous form. There is thus interest in investigating the thermo-acoustic properties associated with various gases infused this way into the brick. We believe thermal-acoustic propagation in such materials will be a nonlinear

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phenomenon and so any theory which allows us to accurately predict the behaviour of a sound wave in a porous material is welcome.

There is much need for accurate theoretical modelling of acoustic wave propagation in porous media. A recent simple nonlinear model has been proposed in a very interesting paper of Jordan [6]. Jordan effectively uses a classical perfect fluid model but adds a term in the momentum equation which is proportional to the velocity, a Darcy-like term (cf. Ref. [7] for an account of Darcy's law). Fellah et al. [8] indicate how transport properties in air-saturated porous media may be measured and Fellah and Depollier [9] show that the equations for mass conservation and momentum in a perfect fluid, in a certain low-frequency approximation lead to a linear system of equations equivalent to the Jordan–Darcy model (in a linearised form).

The work of Jordan [6] is generalised by Ciarletta and Straughan [10] who study his Jordan–Darcy model using acceleration waves. An acceleration wave is a two-dimensional singular surface in a three-dimensional body across which the acceleration suffers a finite discontinuity. The use of acceleration waves and related analyses have proved extremely useful in recent investigations of wave motion in various dispersive and random media, in a variety of thermodynamic states (see e.g. Refs. [11–23]). Since we are able to obtain exactly the wavespeeds and wave amplitudes for both the mechanical and thermal waves, with no approximations, even though we deal with a completely nonlinear theory, we believe this is a main reason why acceleration waves are especially useful.

The object of this paper is to present a more complete model to that of Ciarletta and Straughan [10] in that we include thermal effects which were previously neglected. In fact, the theory developed herein allows, in addition to acoustic wave propagation, for the transmission of heat as a thermal wave. It is increasingly being recognised that such thermal waves (second sound) have a vital role to play in the study of porous media. For example, Meyer [24] has devised an ingenious way of drying a saturated porous material via employment of second sound, Linton-Johnson et al. [25] calculate bulk properties in the porous matrix, and Vadasz et al. [26] discuss how second sound is prominent in nanofluids where many small particles are present. A recent study also shows that second sound is evidently a prominent mechanism for heat transfer in some biological tissues, such tissue also being a porous medium (cf. Ref. [27]).

Various theories have been proposed to allow heat to propagate as a wave of finite speed. For example, the Maxwell–Cattaneo theory (e.g. Refs. [28,26], and the references therein), or the time lag theory (e.g. Refs. [29,30]). However, such models have inherent difficulties when coupled to the other equations of continuum mechanics such as those for the description of fluid or elastic properties, due to the correct representation of time derivatives. For example, Straughan and Franchi [31] and Franchi and Straughan [32] show that entirely different results may be obtained in convection problems depending on which objective derivative is employed. Moreover, D. Graffi (see Ref. [33]) and Morro and Ruggeri [34] show that the coefficient τ in the Maxwell–Cattaneo theory cannot be constant as is usually assumed.

To develop a thermal poroacoustic theory we, therefore, employ the thermodynamics of Green and Laws [35] who use a generalised temperature ϕ instead of the usual temperature θ. Since their approach is developed with a view to producing a rational continuum thermodynamic theory of any solid or fluid it extends naturally to the porous medium application we have in mind and is not subject to any criticism of objective derivatives.

In this paper, we present a theory for acceleration wave propagation in a Darcy porous material which also allows heat to travel with a finite wavespeed. We then show that one can develop a full nonlinear analysis for our model with no approximation whatsoever. Moreover, a precise evolutionary behaviour is predicted for the amplitude of an acceleration wave. This demonstrates that the use of porous materials for constructing modern buildings (horizontal structures and walls) guarantees a large effect on attenuation of acoustic waves where the attenuation may be due to the Darcy effect or to a thermodynamic effect, or a combination of both. Due to the interest in acoustic theory in building materials we believe our results are of value.

2. The model

The model for a perfect fluid employing the thermodynamics of Green and Laws [35] is presented in Ref. [36]. We here include a Darcy term to account for the fact that we are considering heat and sound propagation in a porous medium. The equations of motion are, therefore,

$$\dot{\rho} + \rho v_{i,i} = 0, \tag{1}$$

$$\rho \dot{v}_i = t_{ki,k} - k v_i, \tag{2}$$

$$\rho \dot{\varepsilon} = t_{ki} d_{ik} - q_{i,i}, \tag{3}$$

where we have set the body force and heat supply equal to zero. The quantities $\rho, v_i, t_{ki}, \varepsilon, q_i, d_{ik}$ and k are density, velocity, stress tensor, internal energy, heat flux, symmetric part of the velocity gradient, and the (constant) Darcy coefficient, respectively. Standard indicial notation is used throughout with $_{,i}$ denoting $\partial/\partial x_i$ and a superposed dot denoting the material derivative. The Darcy term $-kv_i$ may be thought of as a friction loss term, or alternatively, one could proceed by considering a mixture of a fluid and a solid, as in Ref. [10], whence this term arises naturally.

The constitutive variables may be taken to be (cf. Ref. [36]), $\rho, \theta, \dot{\theta}$ and $\lambda = \theta_{,i}\theta_{,i}/2$. In terms of the Helmholtz free energy, $\psi = \varepsilon - \eta\phi$ and the generalised (Green–Laws) temperature, ϕ, where η is the entropy, we have the relations (Ref. [36]),

$$\phi = \phi(\theta, \dot{\theta}), \quad \psi = \psi(\rho, \theta, \dot{\theta}, \lambda), \tag{4}$$

$$\eta = -\frac{\partial \psi}{\partial \dot{\theta}} \Big/ \frac{\partial \phi}{\partial \dot{\theta}} = \eta(\rho, \theta, \dot{\theta}, \lambda), \tag{5}$$

$$q_i = -K\theta_{,i}, \tag{6}$$

$$K = \rho\phi \frac{\partial \psi}{\partial \lambda} \Big/ \frac{\partial \phi}{\partial \dot{\theta}} = K(\rho, \theta, \dot{\theta}, \lambda), \tag{7}$$

$$t_{ik} = -p\delta_{ik} - \rho \frac{\partial \psi}{\partial \lambda} \theta_{,i}\theta_{,k}, \tag{8}$$

$$p = \rho^2 \frac{\partial \psi}{\partial \rho} = p(\rho, \theta, \dot{\theta}, \lambda), \tag{9}$$

where p is a pressure and K is a thermal diffusivity. One should note that unlike the isothermal case of Jordan [6] or Ciarletta and Straughan [10], the stress tensor contains the extra piece $\rho(\partial\psi/\partial\lambda)\theta_{,i}\theta_{,k}$, and p is no longer simply a function of ρ.

The residual entropy inequality is

$$-\left(\frac{\partial \psi}{\partial \theta} + \eta \frac{\partial \phi}{\partial \theta}\right)\dot{\theta} + 2K \frac{\partial \phi}{\partial \theta} \frac{\lambda}{\phi} \geqslant 0 \tag{10}$$

and this leads to the relations

$$\left. \left(\frac{\partial \psi}{\partial \theta} + \eta \frac{\partial \phi}{\partial \theta}\right) \right|_E = 0, \tag{11}$$

$$\left. \left(\frac{\partial \eta}{\partial \dot{\theta}} \frac{\partial \phi}{\partial \dot{\theta}}\right) \right|_E - \left. \left(\frac{\partial \eta}{\partial \dot{\theta}}\right) \right|_E \geqslant 0, \tag{12}$$

$$K|_E \geqslant 0, \tag{13}$$

where $f|_E = f(\rho, \theta, \dot{\theta}, \lambda)|_E$ denotes the value of f in thermal equilibrium, i.e. where $\dot{\theta} = \lambda = 0$. In keeping with Green and Laws [35] we suppose $\partial\phi/\partial\theta|_E = 1$.

In view of the ensuing analysis we note that Eq. (3) may be rewritten in the form

$$\begin{aligned} &K\theta_{,ii} + \frac{\partial K}{\partial \rho}\rho_{,i}\theta_{,i} + 2\lambda\frac{\partial K}{\partial \theta} + \frac{\partial K}{\partial \dot{\theta}}\dot{\lambda} + \frac{\partial K}{\partial \lambda}\lambda_{,i}\theta_{,i} \\ &\quad - \rho\left(\frac{\partial \psi}{\partial \theta} + \eta\frac{\partial \phi}{\partial \theta}\right)\dot{\theta} - \rho\phi\frac{\partial \eta}{\partial \rho}\dot{\rho} - \rho\phi\frac{\partial \eta}{\partial \theta}\dot{\theta} - \rho\phi\frac{\partial \eta}{\partial \dot{\theta}}\ddot{\theta} \\ &\quad - \rho\left(\frac{\partial \psi}{\partial \lambda} + \phi\frac{\partial \eta}{\partial \lambda}\right)\dot{\lambda} + \rho\phi\frac{\partial}{\partial \dot{\theta}}\left(\frac{\partial \psi}{\partial \lambda}\Big/\frac{\partial \phi}{\partial \dot{\theta}}\right)\theta_{,i}\theta_{,j}d_{ij} = 0. \end{aligned} \tag{14}$$

We shall suppose ψ and ϕ are such that $\partial\eta/\partial\dot{\theta} > 0$, which is not inconsistent with Eq. (12).

3. Wave amplitudes

Jordan [6] and Ciarletta and Straughan [10] developed acceleration wave analyses for the isothermal version of (1)–(3). Since we are interested in analysing thermodynamic influences on acoustic wave propagation we now develop a complete nonlinear analysis for the full system of Eqs. (1), (2) and (14). Since the theory of acceleration waves is now well known and documented in detail, in, e.g. the research article of Chen [37], we present only the relevant results pertinent to our theory and omit detailed calculations. For system (1), (2) and (14) we define an acceleration wave to be a singular surface $\mathcal{S}$ across which the velocity v_i, the density ρ, and the temperature gradient $\theta_{,i}$ are continuous, while their first and higher derivatives, in general, possess finite discontinuities.

It is possible to develop a general acceleration wave analysis in three-dimensions for Eqs. (1), (2) and (14), i.e. where the acceleration wave is a two-dimensional surface $\mathcal{S}$ moving through a three-dimensional porous body. However, the key physics in connection with acoustic wave propagation is captured by considering a plane wave moving through a three-dimensional body and in this case we can restrict attention to Eqs. (1), (2) and (14), in the one-dimensional picture. In the full three-dimensional scenario the differential geometry involved, cf. the calculations in elasticity in Ref. [37], or those for a perfect fluid in Ref. [36], could well obscure the essential physics we wish to highlight. In one space dimension with $\mathbf{v} = (u, 0, 0)$ and a wave moving in the x-direction, Eqs. (1), (2) and (14) become

$$\rho(u_t + uu_x) = -p_\rho\rho_x - p_\theta\theta_x - p_{\dot{\theta}}\dot{\theta}_x - p_\lambda\lambda_x - (\rho\psi_\lambda\theta_x^2)_x - ku, \tag{15}$$

$$\rho_t + \rho u_x + u\rho_x = 0, \tag{16}$$

$$\begin{aligned} &K\theta_{xx} + K_\rho\rho_x\theta_x + 2\lambda K_\theta + K_{\dot{\theta}}\dot{\lambda} + K_\lambda\lambda_x\theta_x \\ &\quad - \rho(\psi_\theta + \eta\phi_\theta)\dot{\theta} - \rho\phi\eta_\rho\dot{\rho} - \rho\phi\eta_\theta(\theta_t + u\theta_x) \\ &\quad - \rho\phi\eta_{\dot{\theta}}(\theta_{tt} + 2u\theta_{tx} + u_t\theta_x + uu_x\theta_x + u^2\theta_{xx}) \\ &\quad - \rho(\psi_\lambda + \phi\eta_\lambda)(\lambda_t + u\lambda_x) + 2\rho\phi\frac{\partial}{\partial\dot{\theta}}\left(\frac{\psi_\lambda}{\phi_{\dot{\theta}}}\right)u_x\lambda = 0. \end{aligned} \tag{17}$$

Let $+$ denote the region ahead of $\mathcal{S}$ and let $-$ likewise denote the region behind the wave moving in the $+$ direction. The amplitudes $A(t), B(t)$ and $C(t)$ of the acceleration wave may be defined as

$$A(t) = [u_x] = u_x^- - u_x^+ \tag{18}$$

and

$$B(t) = [\rho_x], \quad C(t) = [\theta_{xx}]. \tag{19}$$

While we could develop a complete analysis from Eqs. (15) to (17) for an arbitrary state ahead of $\mathcal{S}$ we have found that the essential features of the Darcy effect (porous medium) and the influence of thermodynamics are yielded by considering a wave moving into a region at rest at constant temperature, i.e. $u^+ \equiv 0$, $\theta^+ \equiv$ constant, $\rho^+ \equiv$ constant (and this renders the outcome of the calculations more transparent).

By taking the jump of Eqs. (15)–(17) and using the Maxwell relation one may now show that the wavespeed V satisfies the fourth-order equation

$$(V^2 - U_T^2)(V^2 - U_M^2) + \kappa V^2 = 0, \tag{20}$$

where $U_T^2 = K/\rho\phi(\partial\eta/\partial\dot{\theta})$, $U_M^2 = \partial p/\partial\rho$ and

$$\kappa = \frac{\partial p}{\partial\dot{\theta}}\frac{\partial\eta}{\partial\rho}\bigg/\frac{\partial\eta}{\partial\dot{\theta}} = -\rho^2\left(\frac{\partial^2\psi}{\partial\dot{\theta}\,\partial\rho}\right)^2\bigg/\frac{\partial\phi}{\partial\dot{\theta}}\frac{\partial\eta}{\partial\dot{\theta}}. \tag{21}$$

The quantities U_T and U_M are the speeds of a thermal wave in a rigid heat conductor and a sound wave in a porous medium (cf. Refs. [38,6,10]). From Eq. (21) we might expect $\kappa<0$ and we suppose this to be the case. Then, Eq. (20) shows that there are two sets of waves, each moving to the right and left with speeds $\pm V_1, \pm V_2$ where

$$V_2^2 < (U_T^2, U_M^2) < V_1^2. \tag{22}$$

Experience with second sound waves (cf. Ref. [28]) and the references therein, suggests the thermal wave will be the slower. Thus, by studying the first wave with speed V_1 we can obtain information on acoustic wave propagation into a porous medium taking into account also temperature-dependent properties.

In fact, we now solve for the amplitudes A, B, C for the wave with speed V_1. This is a straightforward but somewhat involved calculation, cf. the method in Ref. [37]. We present the outcome. One may show that A satisfies the equation

$$\frac{\delta A}{\delta t} + bA + aA^2 = 0, \tag{23}$$

where $\delta/\delta t$ is the intrinsic derivative (i.e. the derivative at the wavefront),

$$\begin{aligned} b = {} & \frac{k}{2\rho}\left\{\frac{p_\rho}{V^2} + \frac{2\phi\eta_{\dot{\theta}}(p_\rho - V^2)}{(K - \rho\phi\eta_{\dot{\theta}}V^2)}\right\}^{-1} \\ & + \frac{(p_\rho - V^2)\{V^2\rho\phi\eta_\theta + (K - \rho\phi\eta_{\dot{\theta}}V^2)p_\theta/V^2 p_{\dot{\theta}}\}}{2\{\rho\phi\eta_{\dot{\theta}}(p_\rho - V^2) + (K - \rho\phi\eta_{\dot{\theta}}V^2)p_\rho/V^2\}} \end{aligned} \tag{24}$$

and

$$\begin{aligned} a = {} & \frac{1}{2\{\rho\phi\eta_{\dot{\theta}}(p_\rho - V^2) + (p_\rho/V^2)(K - \rho\phi\eta_{\dot{\theta}}V^2)\}} \\ & \times\left\{p_{\dot{\theta}}\phi\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial\eta}{\partial\rho}\right) + \frac{(K - \rho\phi\eta_{\dot{\theta}}V^2)}{V^2}\left(2\frac{\partial p}{\partial\rho} + \rho\frac{\partial^2 p}{\partial\rho^2}\right)\right. \\ & + \left(\frac{p_\rho}{V^2} - 1\right)\left\{\left(2 - \frac{3\rho p_{\dot{\theta}\rho}}{p_{\dot{\theta}}}\right)(K - \rho\phi\eta_{\dot{\theta}}V^2)\right. \\ & \left. + 2\rho\frac{\partial K}{\partial\rho} + V^2\{2\rho\phi\eta_{\dot{\theta}} - \rho^2\eta_\rho(\phi_{\dot{\theta}} + 2\dot{\theta}\phi)\}\right\} \\ & + \left(\frac{p_\rho}{V^2} - 1\right)^2\left\{\frac{(K - \rho\phi\eta_{\dot{\theta}}V^2)}{p_{\dot{\theta}}^2}(\rho V^2 p_{\dot{\theta}\dot{\theta}} + \rho p_\lambda + 2\rho^2\psi_\lambda)\right. \\ & \left.\left. + \frac{\rho^2 V^2}{p_{\dot{\theta}}}\{V^2(\phi_{\dot{\theta}}\eta_{\dot{\theta}} + \phi\eta_{\dot{\theta}\dot{\theta}} - 2K_{\dot{\theta}}) + \psi_\lambda + \phi\eta_\lambda\}\right\}\right\}. \end{aligned} \tag{25}$$

The solution to Eq. (23) is

$$A(t) = \frac{A(0)}{e^{bt} + A(0)ab^{-1}(e^{bt} - 1)}. \tag{26}$$

This yields exactly the evolutionary behaviour of the wave amplitude of an acceleration wave moving into a porous medium. The amplitudes $B(t)$ and $C(t)$ then follow from the jumps of Eqs. (15) and (16).

It is worth pointing out that if we take the limit in Eq. (23) of letting the theory approach the isothermal one in which $\dot{\theta}$ and λ are not present then $V^2 \to \partial p/\partial\rho$, and $b \to K/2\rho$, $a \to 3/2 + \rho^2(\partial/\partial\rho)(\rho^{-1}\partial p/\partial\rho)/2\partial p/\partial\rho$. This is in complete agreement with the result found for the isothermal situation by Ciarletta and Straughan [10]. However, Eq. (26) clearly displays the damping effect of the thermodynamic variables. Even if $k \to 0$ (i.e. the porous medium disappears) there is attenuation of the wave amplitude due to the presence of the b term. Eq. (26) allows us to assess the combined effect of the porous medium, via the k term, and the Green and Laws [35] theory. We presently await further experimental results to allow us to construct suitable functions $\phi(\theta, \dot{\theta})$ and $\psi(\rho, \theta, \dot{\theta}, \lambda)$. Once such information is available expression (26) yields exactly the wave amplitude.

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